



Oxford Cambridge and RSA

Monday 13 May 2024 – Afternoon

AS Level Further Mathematics A

Y531/01 Pure Core

Time allowed: 1 hour 15 minutes



You must have:

- the Printed Answer Booklet
- the Formulae Booklet for AS Level Further Mathematics A
- a scientific or graphical calculator

QP

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ ms}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [].
- This document has **8** pages.

ADVICE

- Read each question carefully before you start your answer.

- 1 Use a matrix method to determine the solution of the following simultaneous equations. [4]

$$\begin{aligned} 2x - 3y + z &= 1 \\ x - 2y - 4z &= 40 \\ 5x + 6y - z &= 61 \end{aligned}$$

- 2 In this question you must show detailed reasoning.

(a) Express $\frac{8+i}{2-i}$ in the form $a+bi$ where a and b are real. [2]

(b) Solve the equation $4x^2 - 8x + 5 = 0$. Give your answer(s) in the form $c+di$ where c and d are real. [2]

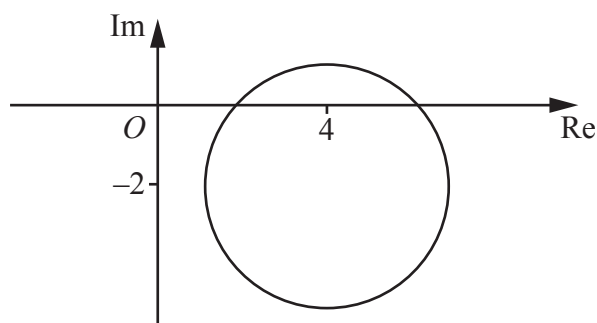
3 (a) (i) Find $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \times \begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix}$. [1]

(ii) State a geometrical relationship between the answer to part (a)(i) and the vectors $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix}$. [1]

(iii) Verify the relationship stated in part (a)(ii). [2]

(b) Find the angle between the vectors $2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $4\mathbf{i} - \mathbf{j} + 8\mathbf{k}$. [3]

- 4 The Argand diagram shows a circle of radius 3. The centre of the circle is the point which represents the complex number $4 - 2i$.



- (a) Use set notation to define the locus of complex numbers, z , represented by points which lie on the circle. [2]

The locus L is defined by $L = \{z : z \in \mathbb{C}, |z - i| = |z + 2|\}$.

- (b) On the Argand diagram in the Printed Answer Booklet, sketch and label the locus L . [2]

You are given that the locus $\{z : z \in \mathbb{C}, \arg(z - 1) = \frac{1}{4}\pi, \operatorname{Re}(z) = 3\}$ contains only one number.

- (c) Find this number. [2]

- 5 The line through points $A(8, -7, -2)$ and $B(11, -9, 0)$ is denoted by L_1 .

- (a) Find a vector equation for L_1 . [2]

- (b) Determine whether the point $(26, -19, -14)$ lies on L_1 . [2]

The line L_2 passes through the origin, O , and intersects L_1 at the point C . The lines L_1 and L_2 are perpendicular.

- (c) By using the fact that C lies on L_1 , find a vector equation for L_2 . [4]

- (d) Hence find the shortest distance from O to L_1 . [2]

- 6 You are given that $\mathbf{A} = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$ where a is a constant.

Prove by induction that $\mathbf{A}^n = \begin{pmatrix} 1 & an \\ 0 & 1 \end{pmatrix}$ for all integers $n \geq 1$. [5]

7 In this question you must show detailed reasoning.

The roots of the equation $2x^3 - 3x^2 - 3x + 5 = 0$ are α , β and γ .

By considering $(\alpha + \beta + \gamma)^2$ and $(\alpha\beta + \beta\gamma + \gamma\alpha)^2$, determine a cubic equation with integer coefficients whose roots are $\frac{\alpha\beta}{\gamma}$, $\frac{\beta\gamma}{\alpha}$ and $\frac{\gamma\alpha}{\beta}$. [6]

8 Three transformations, T_A , T_B and T_C , are represented by the matrices **A, **B** and **C** respectively.**

You are given that $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

(a) Find the matrix which represents the inverse transformation of T_A . [1]

(b) By considering matrix multiplication, determine whether T_A followed by T_B is the same transformation as T_B followed by T_A . [2]

Transformations R and S are each defined as being the result of successive transformations, as specified in the table.

Transformation	First transformation	followed by
R	T_A followed by T_B	T_C
S	T_A	T_B followed by T_C

(c) Explain, using a property of matrix multiplication, why R and S are the same transformations. [2]

A quadrilateral, Q , has vertices D , E , F and G in anticlockwise order from D . Under transformation R, Q 's image, Q' , has vertices D' , E' , F' and G' (where D' is the image of D , etc). The area of Q , in suitable units, is 5.

You are given that $\det \mathbf{C} = a^2 + 1$ where a is a real constant.

(d) (i) Determine the order of the vertices of Q' , starting anticlockwise from D' . [2]

(ii) Find, in terms of a , the area of Q' . [1]

(iii) Explain whether the inverse transformation for R exists. Justify your answer. [2]

9 In this question you must show detailed reasoning.

You are given that a is a real root of the equation $x^4 + x^3 + 3x^2 - 5x = 0$.

You are also given that $a + 2 + 3i$ is one root of the equation

$$z^4 - 2(1 + a)z^3 + (21a - 10)z^2 + (86 - 80a)z + (285a - 195) = 0.$$

Determine all possible values of z .

[8]

END OF QUESTION PAPER

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